Week 7
MATH 34B
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8. Find the general solution of the equation $y' = 3(1-y)$.

$$\frac{dy}{dx} = 3(1-y)$$

$$\int \frac{dy}{1-y} = \int 3\,dx$$

$$-\ln|1-y| = 3x + C$$

$$\ln|1-y| = -3x + C$$

$$1-y = Ce^{-3x}$$

$$y = 1 - Ce^{-3x}$$

10. Find the solution of the equation $h = \cancel{0.4(4h)}$ $0.4(4-h)$ with initial condition $h(0) = 1$.

$$\frac{dh}{dt} = 0.4(4-h)$$

$$\int \frac{dh}{4-h} = \int 0.4\,dt$$

$$-\ln(4-h) = 0.4t + C$$

$$\ln(4-h) = -0.4t + C$$

$$4-h = Ce^{-0.4t}$$

$$\Rightarrow h = 4 - Ce^{-0.4t}$$

$\Rightarrow h(0) = 1 \Rightarrow C = 3$

$\Rightarrow h = 4 - 3e^{-0.4t}$

13. Solve the equation $y = \cancel{156y}$ $15-6y$. with initial condition $y(0) = 0$.

How quickly was the total income of the entire population rising in 1997?

$$\frac{dy}{dt} = 15-6y$$

$$\int \frac{dy}{15-6y} = \int dt$$

$$-\frac{1}{6}\ln|15-6y| = t + C$$

$$\ln|15-6y| = -6t + C.$$

$$15-6y = Ce^{-6t}$$

$$y = \frac{15}{6} - Ce^{-6t}.$$

$$y(0)=0 \Rightarrow C = 15/6.$$

$$y = \frac{15}{6} - \frac{15}{6}e^{-6t}.$$

18. A cup of coffee was made at a temperature of $90°C$ and cools according to Newton's law of cooling. The room temperature is $30°C$. If the temperature of the coffee 20 minutes after being made was $40°C$.

    (a) What was the temperature of the coffee 5 minutes after being made?

    (b) When was the temperature of the coffee $75°C$?

$$\frac{dT}{dt} \sim T-30$$

$$\frac{dT}{dt} = k(T-30)$$

$$\int \frac{dT}{T-30} = \int k\,dt$$

$$\ln|T-30| = kt + C$$

$$T-30 = Ce^{kt}.$$

$$T = Ce^{kt} + 30.$$

$$T(0) = 90$$

$$\Rightarrow 90 = C + 30.$$

$$\Rightarrow T = 60e^{kt} + 30$$

$$T(20) = 40$$

$$\ldots \Rightarrow k = \ln(1/6)/20.$$

So, $T = 60e^{kt} + 30$, where

a) $T(5) = 60e^{k\cdot 5} + 30$

b) $75 = 60e^{kt} + 30$

$$45 = 60e^{kt}$$

$$\ldots \quad t = \frac{\ln(3/4)}{k}.$$

21. Find the partial solutions to the DE $\frac{dy}{dx} = (x-2)e^{-2y}$ satisfying $y(2) = \ln(2)$.

$$\frac{dy}{dx} = (x-2)e^{-2y}$$

$$\int e^{2y}\,dy = \int (x-2)\,dx$$

$$\frac{1}{2}e^{2y} = \frac{x^2}{2} - 2x + C$$

$$e^{2y} = x^2 - \cancel{4}x + C$$

$$2y = \ln(x^2-4x+C)$$

$$y = \frac{1}{2}\ln(x^2-4x+C)$$

$$y(2) = \ln 2$$

$$\Rightarrow \ln 2 = \frac{1}{2}\ln(2^2-4\cdot 2+C)$$

$$2 = e^{\frac{1}{2}\ln(\sim\sim)}$$

$$= (2^2-8+C)^{\frac{1}{2}}$$

$$\Rightarrow 4 = 4-8+C$$

$$\Rightarrow C = 8.$$

so, $y = \frac{1}{2}\ln(x^2-4x+8)$

25. Find the equation for the curve satisfying $\frac{dy}{dx} = 90yx^{17}$ where the y-intercept is 6.

$$\frac{dy}{dx} = 90yx^{17}$$

$$\int \frac{dy}{y} = \int 90x^{17}$$

$$\ln|y| = \frac{90x^{18}}{18} = 5x^{18} + C$$

$$y = Ce^{5x^{18}}$$

$$y(0) = 6 \Rightarrow C = 6$$

$$y = 6e^{5x^{18}}.$$

27. Suppose $Q = Ce^{kt}$, and Q satisfies $\frac{dQ}{dt} = -0.05Q$. What does this tell you about k and C?

$$\frac{dQ}{dt} = Cke^{kt} = -0.05Ce^{kt}$$

$e^{kt} \neq 0$ for any t, so...

$$Ck = -0.05C.$$

So... $k = -0.05$, C = anything

OR $C = 0$, k = anything.

29. Given $\frac{dy}{dt} = 100 - y$, find y when

(a) $y(0) = 35$

(b) $y(0) = 135$.

$$\int \frac{dy}{100-y} = \int dt$$

$$-\ln|100-y| = t + C$$

$$\ln|100-y| = -t + C$$

$$100 - y = Ce^{-t}$$

$$y = 100 - Ce^{-t}$$

a) $y(0) = 35$

$35 = 100 - Ce^{0}$

$\Rightarrow C = 65$

$\Rightarrow y(t) = 100 - 65e^{-t}$

b) $y(0) = 135$

$135 = 100 - Ce^{0}$

$35 = -C\cancel{e^{0}}$

$-C = 35$

$\Rightarrow y(t) = 100 + 35e^{-t}$